

## Three-particle contributions to the renormalisation of $B$ -meson light-cone distribution amplitudes

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## Three-particle contributions to the renormalisation of $B$ -meson light-cone distribution amplitudes

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**ABSTRACT:** We study light-cone distribution amplitudes of heavy-light systems, such as a  $B$ -meson. By an explicit computation, we determine how two-parton distribution amplitudes mix with three-parton ones at one loop:  $\phi_+$  is shown to mix only into itself, whereas  $\phi_-$  mixes with the difference of three-parton distribution amplitudes  $\Psi_A - \Psi_V$ . We determine the corresponding anomalous dimension and we check the gauge independence of our result by considering a general covariant gauge. Finally, we comment on some implications of our result for phenomenological models of these distribution amplitudes.

**KEYWORDS:** B-Physics, NLO Computations, Renormalization Group, QCD

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## 1 Introduction

$B$  physics provides outstanding opportunities to test the Standard Model. At the end of the era of  $B$ -factories, a remarkable sample of processes has been measured accurately. On the theoretical side, the dynamics of heavy-light systems is naturally dominated by two scales: the heavy-quark mass  $m_b$  corresponding to hard contributions and a hadronic scale  $\Lambda$  related to soft, hadronic, contributions. It is therefore natural to deal with  $B$ -mesons in the framework of the heavy-quark expansion, expanding observables in powers of  $1/m_b$ , and to attempt to factorise hard and soft dynamics.

This has been progressively achieved within the frameworks of QCD factorisation [1–3] and Soft-Collinear Effective Theory [4–8]. The hadronic input collecting soft physics are not only the form factors, but also the light-cone distribution amplitudes, defined generally as the matrix element of a non-local operator along a light-like direction. This quantity provides the amplitude of probability of finding partons inside a hadron with given fractions of the hadron momentum. For light hadrons, one can define a twist, corresponding to the

power of the hard scale to which the distribution amplitude will contribute to a given process, which is related to the number of partons and the Lorentz structure of interest [9]. For instance, the leading-twist distribution amplitude of the pion can be defined as:

$$\langle \pi(p) | \bar{q}(z_2)[z_2, z_1] \gamma_\mu \gamma_5 q(z_1) | 0 \rangle = -i f_\pi p_\mu \int_0^1 dx e^{i(xp \cdot z_2 + \bar{x}p \cdot z_1)} \phi_\pi(x) \quad (1.1)$$

where  $(z_2 - z_1)^2 = 0$ ,  $\bar{x} = 1 - x$ , and the path-ordered exponential, ensuring the gauge invariance of the expression, reads:

$$[z_2, z_1] = P \exp \left[ i g_s \int_{z_1}^{z_2} dy_\mu A^\mu(y) \right] \quad (1.2)$$

This distribution amplitude corresponds to picking up a pair of valence quark and anti-quark in the pion, carrying a fraction  $x$  and  $\bar{x}$  of the pion momentum respectively.

In the early days of QCD and the parton model, distribution amplitudes of light mesons were identified as key tools to analyse exclusive processes at high energy through factorisation, and they have been analysed extensively [10–12]. Phenomenologically, sum rules and lattice have been used to determine the values of their lowest moments [13–16]. Mathematically, the structure of the distribution amplitudes was described by exploiting properties of the conformal group, providing a geometrical definition of twist compatible with the phenomenological one in the case of light mesons [9].

The case of heavy-light mesons has been discussed only recently [17, 18], mainly due to their importance in relation with  $B$ -physics, as shown in the case of non-leptonic decays [1–3], semileptonic decays [19, 20], radiative decays [21–25]. It is possible to define two two-parton distribution amplitudes  $\phi_+$  and  $\phi_-$  from the most simple non-local matrix element:

$$\langle 0 | \bar{q}_\beta(z)[z, 0](h_v)_\alpha(0) | B(p) \rangle = -i \frac{\hat{f}_B(\mu)}{4} \left[ (1 + \not{v}) \left( \tilde{\phi}_+(t) + \frac{\not{z}}{2t} [\tilde{\phi}_-(t) - \tilde{\phi}_+(t)] \right) \gamma_5 \right]_{\alpha\beta} \quad (1.3)$$

with the usual definitions of Heavy Quark Effective Theory (HQET) for the velocity  $v = p/M_B$ , the heavy-quark projection  $h_v$  and the decay constant  $\hat{f}_B$ . The Fourier transforms of the distribution amplitudes, depending on  $t = v \cdot z$  read:

$$\tilde{\phi}_\pm(t) = \int_0^\infty d\omega e^{-i\omega t} \phi_\pm(\omega). \quad (1.4)$$

It turns out that only one distribution amplitude,  $\phi_+$ , enters most of the computations considered in the framework of factorisation (non-leptonic decays,  $B \rightarrow V\gamma$ ,  $B \rightarrow \gamma\ell\nu$ ) at leading order in  $1/m_B$ . In cases where at least one of the outgoing particles has not a light-like momentum, factorisation may still hold, but the formula involves the two distribution amplitudes of the  $B$  meson. This is in particular the case for  $B \rightarrow V\gamma^*$ , which has an important potential to test the Standard Model [26, 27].

Another interesting place where the distribution amplitudes naturally occur is light-cone sum rules. These sum rules allow in particular for a determination of form factors of phenomenological relevance such as  $B \rightarrow \pi$ ,  $B \rightarrow \rho$ . The purpose of sum rules is to reexpress those form factors in terms of an integral of the distribution amplitude of one

of the external mesons with a kernel resulting from the expansion of a properly chosen correlator along the light-cone. Recently, sum rules with interesting properties have been proposed to relate these form factors with  $B$  meson distribution amplitudes. In this context,  $\phi_-$  plays a dominant role and its modeling can improve the determination of the form factors [28–31].

One can also define distribution amplitudes beyond the two-parton level. Interestingly, one can exploit quark equations of motion in order to relate two-parton and three-parton distribution amplitudes, which are defined in the case of heavy-light mesons through:

$$\begin{aligned} & \langle 0 | \bar{q}_\beta(z) [z, uz] g G_{\mu\nu}(uz) z^\nu [uz, 0] (h_v)_\alpha(0) | B(p) \rangle \\ &= \frac{\hat{f}_B(\mu) M}{4} \left[ (1 + \not{v}) \left[ (v_\mu \not{z} - t \gamma_\mu) \left( \tilde{\Psi}_A(t, u) - \tilde{\Psi}_V(t, u) \right) - i \sigma_{\mu\nu} z^\nu \tilde{\Psi}_V(t, u) \right. \right. \\ & \quad \left. \left. - z_\mu \tilde{X}_A(t, u) + \frac{z_\mu \not{z}}{t} \tilde{Y}_A(t, u) \right] \gamma_5 \right]_{\alpha\beta} \end{aligned} \tag{1.5}$$

A good model for the distribution amplitudes of  $\phi_+$  and  $\phi_-$  must embed as many theoretical constraints as possible. Not much can be said for sure, apart from sum rule estimates [32, 33], the expected behaviour at the origin ( $\phi_+(\omega) \sim \omega$  and  $\phi_-(\omega) \sim 1$  for  $\omega \rightarrow 0$ ), and the properties of these quantities under renormalisation, which can be derived from perturbation theory. One must notice that differences between light-meson and heavy-light meson in terms of the renormalisation properties [33] exist: the limit in one case is the chiral limit, which modifies only long-distance properties, whereas the heavy-quark limit affects short-distance features of the theory (the UV structure of HQET is qualitatively different from QCD). This explains the non-commutation of the heavy-quark limit with the light-cone limit (contrary to the chiral limit). In the case of heavy-quark distribution amplitude, there is no equivalent of Gegenbauer moments of light-meson distribution amplitudes, which mix only into themselves under renormalisation.

The RGE behaviour of  $\phi_+$  and  $\phi_-$  has been investigated in refs. [34] and [35]. It was shown in particular that RGE generates a radiative tail, leading to a divergence of positive moments of these quantities. In particular, there is no absolute normalisation of  $\phi_\pm$  to 1 from its zeroth moment, so that a naive partonic interpretation like in the pion case is not possible after renormalisation. To obtain the first inverse moments of  $\phi_+$  and  $\phi_-$ , which are relevant phenomenologically, one needs the knowledge of the whole distribution amplitude. But the previous behaviours and models were derived in the two-parton approximation, even though the equation of motions indicate the potential mixing with three-parton distribution amplitudes.

The goal of this paper is to understand the renormalisation of these objects at the first nontrivial order of the strong coupling constant, including the contribution from three-parton distribution amplitudes. This task requires us to consider a three-parton external state (a quark, an antiquark and a gluon). In section 2, we recall the behaviour of  $\phi_+$  and  $\phi_-$  under renormalisation as determined from a two-parton external state. In section 3, we give the one-loop diagrams and their ultraviolet divergences in the case of a three-parton external state. In section 4, we determine the mixing of  $\phi_+$  and  $\phi_-$  with three-parton distribution amplitudes at one loop, and we show that there is no mixing in the case of  $\phi_+$ ,

whereas  $\phi_-$  mixes with the difference  $\Psi_A - \Psi_V$ . In section 4, we extend our calculation, performed in the Feynman gauge, to a general covariant gauge, which provides a check of the gauge independence of our results (more detailed results are given in appendix A). In section 5, we make a few comments before concluding.

## 2 Two-parton $B$ -meson distribution amplitudes

We set up our framework by introducing notation and definitions, and we recall the results obtained on the renormalisation of the  $B$ -meson distribution amplitudes. We define light-cone directions by two vectors

$$n_+^2 = n_-^2 = 0 \quad n_+ \cdot n_- = 2 \quad v = (n_+ + n_-)/2 \quad (2.1)$$

so that an arbitrary vector can be projected as

$$q_\mu = (n_+ \cdot q) \frac{n_{+,\mu}}{2} + (n_- \cdot q) \frac{n_{-,\mu}}{2} + q_{\perp\mu} = q_+ \frac{n_{+,\mu}}{2} + q_- \frac{n_{-,\mu}}{2} + q_{\perp\mu} \quad (2.2)$$

The computation of the renormalisation properties of the distribution amplitudes requires us to consider matrix elements of the relevant operators

$$O_+^H(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \langle 0 | \bar{q}(z) [z, 0] \not{n}_+ \Gamma h_v(0) | H \rangle \quad (2.3)$$

$$O_-^H(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \langle 0 | \bar{q}(z) [z, 0] \not{n}_- \Gamma h_v(0) | H \rangle \quad (2.4)$$

$$O_3^H(\omega, \xi) = \frac{1}{(2\pi)^2} \int dt e^{i\omega t} \int du e^{i\xi u t} \langle 0 | \bar{q}(z) [z, uz] g_s G_{\mu\nu}(uz) z^\nu [uz, 0] \Gamma h_v(0) | H \rangle \quad (2.5)$$

with  $z$  parallel to  $n_+$ , i.e.  $z_\mu = t n_{+,\mu}$ ,  $t = v \cdot z = z_-/2$  and the path-ordered exponential in the  $n_+$  direction:

$$[z, 0] = P \exp \left[ i g_s \int_0^z dy_\mu A^\mu(y) \right] \quad (2.6)$$

$$= 1 + i g_s \int_0^1 d\alpha z_\mu A^\mu(\alpha z) - g_s^2 \int_0^1 d\alpha \int_0^\alpha d\beta z_\mu z_\nu A^\mu(\alpha z) A^\nu(\beta z) + \dots \quad (2.7)$$

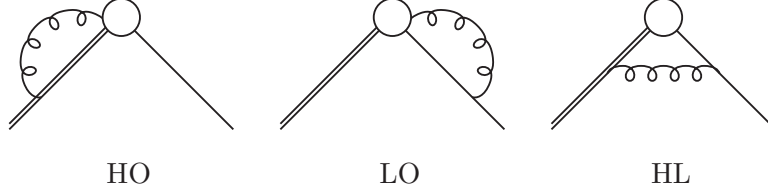
We define the different distribution amplitudes in momentum space through their Fourier transforms:

$$\phi_\pm(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \tilde{\phi}_\pm(t) \quad F(\omega, \xi) = \frac{1}{(2\pi)^2} \int dt \int du e^{i(\omega+u\xi)t} \tilde{F}(t, u) \quad (2.8)$$

where  $F = \Psi_V, \Psi_A, X_A, Y_A$ .

The renormalisation group equation of  $\phi_+$  and  $\phi_-$  can be determined by computing the mixing terms  $Z$  defined in the following expression (since we work in the chiral limit, we omit mixing terms between  $\phi_+$  and  $\phi_-$  which were shown in ref. [35] to be proportional to the mass of the light-quark.)

$$O_\pm^{\text{H,ren}}(\omega, \mu) = \int d\omega' Z_\pm^{-1}(\omega, \omega'; \mu) O_\pm^{\text{H,bare}}(\omega') + \int d\omega' d\xi' Z_{\pm,3}^{-1}(\omega, \omega', \xi'; \mu) O_3^{\text{H,bare}}(\omega', \xi') + \dots \quad (2.9)$$



**Figure 1.** Diagrams evaluated to determine the renormalisation constant  $Z_{\pm}$  using a two-parton external state (the two letters describe the ends of the gluon line: Heavy quark, Light quark or Operator, i.e. the path-ordered exponential).

where the ellipsis involves matrix elements of operators corresponding to a higher number of partons (a similar equation for operators related to a higher number of partons instead of  $O_{\pm}$  could be written, with operators corresponding to an arbitrary number of partons on the right-hand side). The behaviour under renormalisation being a short-distance property, any choice of  $H$  is allowed in principle. For instance,  $Z_{\pm}$  were computed in refs. [34, 35] using a two-parton external state with on-shell quarks  $H = h(p)\bar{q}(k)$ , for which one obtains at leading order:

$$O_{\pm}^H(\omega) = \delta(\omega - k_+) \bar{v}\not{l}_{\pm}\Gamma u \quad O_3^H(\omega, \omega') = 0 \quad (2.10)$$

Computing NLO terms provides  $Z_{\pm}$  in eq. (2.9). However convenient, this choice of external state prevents us from determining  $Z_{\pm,3}$  describing the mixing between  $O_{\pm}$  and  $O_3$ , since  $O_3^H$  vanishes then.

One can still write down the RGE restricted to  $\phi_{\pm}$  as

$$\frac{d}{d \log \mu} \phi_{\pm}(\omega, \mu) = - \int_0^{\infty} d\omega' \gamma_{\pm}(\omega, \omega', \mu) \phi_{\pm}(\omega', \mu) + \dots \quad (2.11)$$

$$\gamma_{\pm}(\omega, \omega', \mu) = - \int d\tilde{\omega} \frac{dZ_{\pm}^{-1}(\omega, \tilde{\omega}, \mu)}{d \log \mu} Z_{\pm}(\tilde{\omega}, \omega', \mu) - \gamma_F(\alpha_s) \delta(\omega - \omega') \quad (2.12)$$

where  $\gamma_F$  is due to the normalisation of the distribution amplitudes which involves the (renormalisation-scale dependent) HQET decay constant.

Working in dimensional regularisation with  $d = 4 - 2\varepsilon$  dimensions, in the Feynman gauge and in the  $\overline{\text{MS}}$  scheme, one obtains from the three diagrams shown in figure 1 [34, 35]:

$$Z_{\pm}|_{\text{HO}} = \frac{\alpha_s C_F}{4\pi} \times 2 \times \frac{1}{\varepsilon} \int_0^{\infty} \frac{dl_+}{l_+} \left( \frac{l_+^2}{\mu^2} \right)^{-\varepsilon} [\delta(\omega - \omega' - l_+) - \delta(\omega - \omega')] \quad (2.13)$$

$$Z_{\pm}|_{\text{LO}} = \frac{\alpha_s C_F}{4\pi} \times (-2) \times \frac{1}{\varepsilon} \int_0^{\omega'} \frac{dl_+ l_+ - \omega'}{l_+ \omega'} [\delta(\omega' - \omega - l_+) - \delta(\omega' - \omega)] \quad (2.14)$$

$$Z_{-}|_{\text{HL}} = \frac{\alpha_s C_F}{4\pi} \times 2 \times \frac{1}{\varepsilon} \int_0^{\omega'} \frac{dl_+}{\omega'} \delta(\omega' - \omega - l_+) \quad Z_{+}|_{\text{HL}} = 0 \quad (2.15)$$

Only the diagram where the gluon line connects the two external legs differs for  $\phi_+$  and  $\phi_-$ . In addition, one has contributions from the wave-function renormalisation of the external legs:

$$Z_{\pm}|_{\text{wfr-H}} = Z_h^{1/2} = \frac{\alpha_s C_F}{4\pi} \times \frac{1}{\varepsilon} \delta(\omega - \omega') \quad (2.16)$$

$$Z_{\pm}|_{\omega_{fr-L}} = Z_q^{1/2} = \frac{\alpha_s C_F}{4\pi} \times \left(-\frac{1}{2}\right) \times \frac{1}{\varepsilon} \delta(\omega - \omega') \quad (2.17)$$

so that  $Z_{\pm}$  is  $\delta(\omega - \omega')$  corrected at one loop by the sum of eqs. (2.13)–(2.17).

This leads to the following anomalous dimensions:

$$\gamma_+^{(1)} = \left(\Gamma_{\text{cusp}}^{(1)} \log \frac{\mu}{\omega} + \gamma^{(1)}\right) \delta(\omega - \omega') - \Gamma_{\text{cusp}}^{(1)} \omega \left(\frac{\theta(\omega' - \omega)}{\omega'(\omega' - \omega)} + \frac{\theta(\omega - \omega')}{\omega(\omega - \omega')}\right)_+ \quad (2.18)$$

$$\gamma_-^{(1)} = \gamma_+^{(1)} - \Gamma_{\text{cusp}}^{(1)} \frac{\theta(\omega' - \omega)}{\omega'} \quad (2.19)$$

with

$$\Gamma_{\text{cusp}}^{(1)} = 4C_F \quad \gamma^{(1)} = -2C_F \quad \gamma_F^{(1)} = -3C_F \quad (2.20)$$

Quantities with a superscript (1) must be multiplied by  $\frac{\alpha_s}{4\pi}$ . The anomalous dimension is of the Sudakov type, which is related to the fact that the operators of interest can be seen as containing two Wilson lines, one from the heavy quark along the  $v$  direction (representing the interaction of soft gluons with  $h_v$ ) from  $-\infty$  to 0, linked with another one along the  $n_+$  direction from 0 to  $z$ . The presence of a cusp between the two Wilson lines is responsible for the appearance of a Sudakov-like behaviour of the anomalous dimension [34, 36, 37].

### 3 One-loop computation for three-particle external state

We perform the same computation, taking as an external state  $H = h(p)g(\epsilon, q)\bar{q}(k)$  containing three partons. We compute the diagrams at one loop, using dimensional regularisation, and we pay a special attention to separating  $\varepsilon$ -poles related to UV divergences and IR divergences carefully. We then identify the  $\varepsilon$ -poles corresponding to UV divergences as part of the renormalisation function in:

$$O_{\pm}^{\text{H,bare}}(\omega) = \int d\omega' Z_{\pm}(\omega, \omega'; \mu) O_{\pm}^{\text{H,ren}}(\omega', \mu) + \int d\omega' d\xi' Z_{\pm,3}(\omega, \omega', \xi'; \mu) O_3^{\text{H,ren}}(\omega', \xi', \mu) \quad (3.1)$$

Subtracting the contribution from  $Z_{\pm} \otimes O_{\pm}^H$  will yield the mixing term  $Z_{\pm,3}$  between two and three-particle distribution amplitudes.

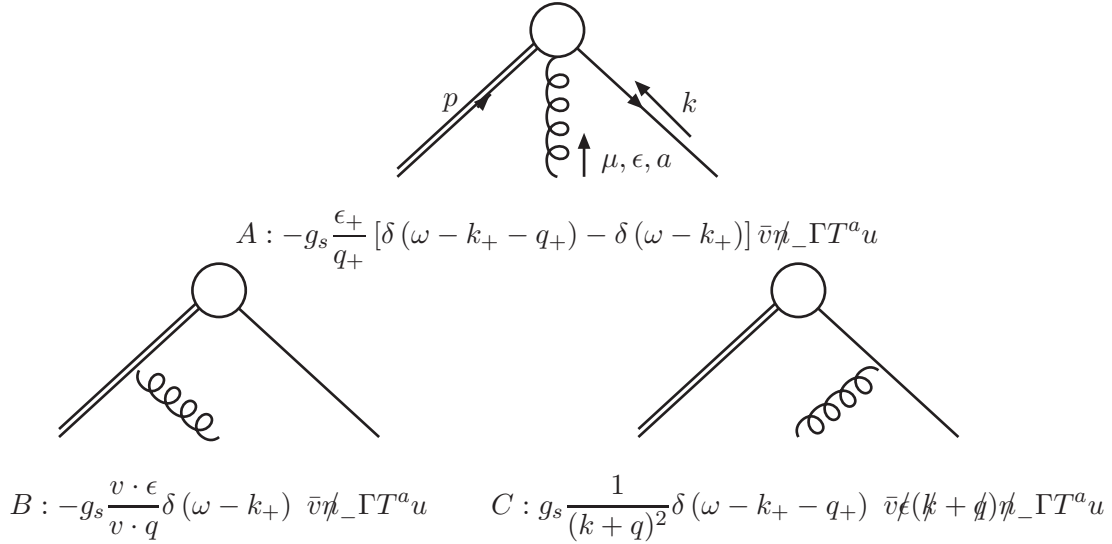
At leading order, we obtain three different contributions, denoted  $A, B, C$ , for the matrix element of  $O_{\pm}$ , shown in figure 2. The expression for  $C$  can be simplified using the following relations

$$\epsilon \cdot q = 0 \quad \not{p}u = u \quad \bar{v}\not{k} = 0 \quad (3.2)$$

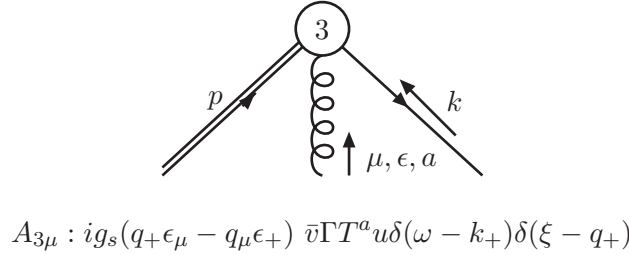
For  $O_3$ , we have a leading-order expression indicated in figure 3.

One can easily spell out the diagrams for  $O_{\pm}$  at one loop by taking the above three diagrams  $ABC$ , and adding a new gluon line in all possible ways (our naming scheme reflects this idea). Diagram  $A$  yields the diagrams in figure 4, whereas the other diagrams coming from  $B$  and  $C$  are show in figure 5. Let us notice the presence of redundant diagrams, namely  $(B24) = (A12)$ ,  $(C24) = (A23)$ ,  $(B25) = (B12)$ ,  $(C25) = (C12)$ .





**Figure 2.** The three leading-order contributions to the matrix element of  $O_{\pm}$  with a three-parton external state.



**Figure 3.** Leading-order contribution to the matrix element of  $O_{3\mu}$  with a three-parton external state.

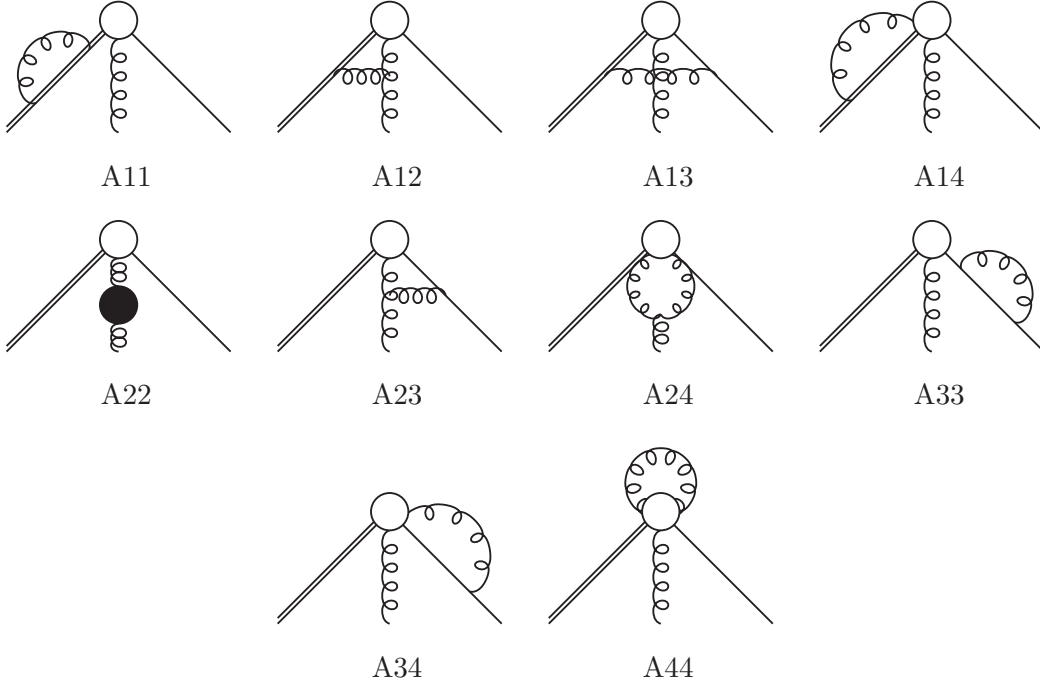
### 3.1 Common contributions

Since we are interested in the renormalisation properties of the distribution amplitudes in the  $\overline{\text{MS}}$  scheme, we quote here only the poles in  $\epsilon$  defined as  $d = 4 - 2\epsilon$  corresponding to ultraviolet divergences (we discuss our integration procedure on one example explicitly in appendix A). For the integrals going up to infinity, we keep the expression of the kernels before picking up the pole in  $\epsilon$ , since the integration may give rise to double poles in the expression of  $\gamma_{\pm,3}$ , related to Sudakov logarithms. We do not give the expressions corresponding to the wave-function renormalisation of the external legs, i.e., (11), (22), (33).

The following diagrams yield contributions of the same form for both distribution amplitudes:

$$(A12) = -\frac{\alpha_s}{4\pi} C_A \left(\frac{1}{2}\right) g_s \epsilon_+ \bar{v} \not{\epsilon}_{\pm} \Gamma T^a u \frac{1}{\epsilon} \int_{-q_+}^0 dl_+ \frac{1}{q_+ + l_+} [\delta(\omega - k_+ + l_+) - \delta(\omega - k_+)]$$

$$(A14) = -\frac{\alpha_s}{4\pi} (g_s \epsilon_+) \bar{v} \not{\epsilon}_{\pm} \Gamma T^a u \frac{1}{\epsilon} \int_0^{\infty} dl_+ \left(\frac{l_+^2}{\mu^2}\right)^{-\epsilon}$$



**Figure 4.** One-loop diagrams obtained from diagram A.

$$\begin{aligned}
 & \times \left[ \frac{2C_F}{l_+ q_+} \{ [\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+ - q_+)] + [\delta(\omega - k_+) - \delta(\omega - k_+ - l_+)] \} \right. \\
 & \left. - \frac{C_A}{q_+} \left\{ \frac{1}{l_+ + q_+} [\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+)] - \frac{1}{l_+} [\delta(\omega - k_+ - l_+) - \delta(\omega - k_+)] \right\} \right] \\
 (A24) = & -\frac{\alpha_s}{4\pi} C_A \frac{(g_s \epsilon_+)}{2} \bar{v} \not{\epsilon}_\pm \Gamma T^a u \frac{1}{\epsilon} \int_{-q_+}^0 dl_+ \frac{2l_+ + q_+}{q_+(q_+ + l_+)} \\
 & \times \left[ \frac{1}{l_+} [\delta(\omega - k_+ + l_+) - \delta(\omega - k_+)] + \frac{1}{q_+} [\delta(\omega - k_+ - q_+) - \delta(\omega - k_+)] \right] \\
 (A34) = & \frac{\alpha_s}{4\pi} (g_s \epsilon_+) \bar{v} \not{\epsilon}_\pm \Gamma T^a u \frac{1}{\epsilon} \int_0^{k_+} \frac{dl_+}{l_+} \frac{k_+ - l_+}{k_+} \\
 & \times \left[ \frac{2C_F}{q_+} \{ [\delta(\omega - k_+ - q_+) - \delta(\omega - k_+ - q_+ + l_+)] + [\delta(\omega - k_+ + l_+) - \delta(\omega - k_+)] \} \right. \\
 & \left. - C_A \left\{ \frac{1}{l_+ + q_+} [\delta(\omega - k_+ - q_+) - \delta(\omega - k_+ + l_+)] \right. \right. \\
 & \left. \left. - \frac{1}{q_+} [\delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ + l_+)] \right\} \right] \\
 (A44) = & 0
 \end{aligned}$$

For diagrams of *B*-type, we obtain:

$$\begin{aligned}
 (B12) &= 0 \\
 (B13) &= 0 \\
 (B14) &= 0
 \end{aligned}$$

$$\begin{aligned}
 (B15) &= -\frac{\alpha_s}{4\pi} g_s \frac{1}{\varepsilon} (C_A - 2C_F) \frac{v \cdot \epsilon}{v \cdot q} \delta(\omega - k_+) \bar{v} \not{k}_\pm \Gamma T^a u \\
 (B34) &= B \otimes Z_\pm|_{\text{LO}} \\
 (B35) &= B \otimes Z_\pm|_{\text{LH}} \\
 (B44) &= 0 \\
 (B45) &= B \otimes Z_\pm|_{\text{HO}} \\
 (B55) &= B \times Z_h
 \end{aligned}$$

For diagrams of  $C$ -type, we obtain:

$$\begin{aligned}
 (C12) &= \frac{\alpha_s}{4\pi} C_A \frac{3}{2} \frac{1}{(k+q)^2} \bar{v} \not{k} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u \frac{1}{\varepsilon} \delta(\omega - k_+ - q_+) \\
 (C15) &= \frac{\alpha_s}{4\pi} (C_F - C_A/2) \frac{1}{(k+q)^2} \bar{v} \not{k} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u \frac{1}{\varepsilon} \delta(\omega - k_+ - q_+) \\
 (C34) &= C \otimes Z_\pm|_{\text{HO}} \\
 (C35) &= C \otimes Z_\pm|_{\text{HL}} \\
 (C44) &= 0 \\
 (C55) &= C \times Z_q
 \end{aligned}$$

### 3.2 $\phi_+$

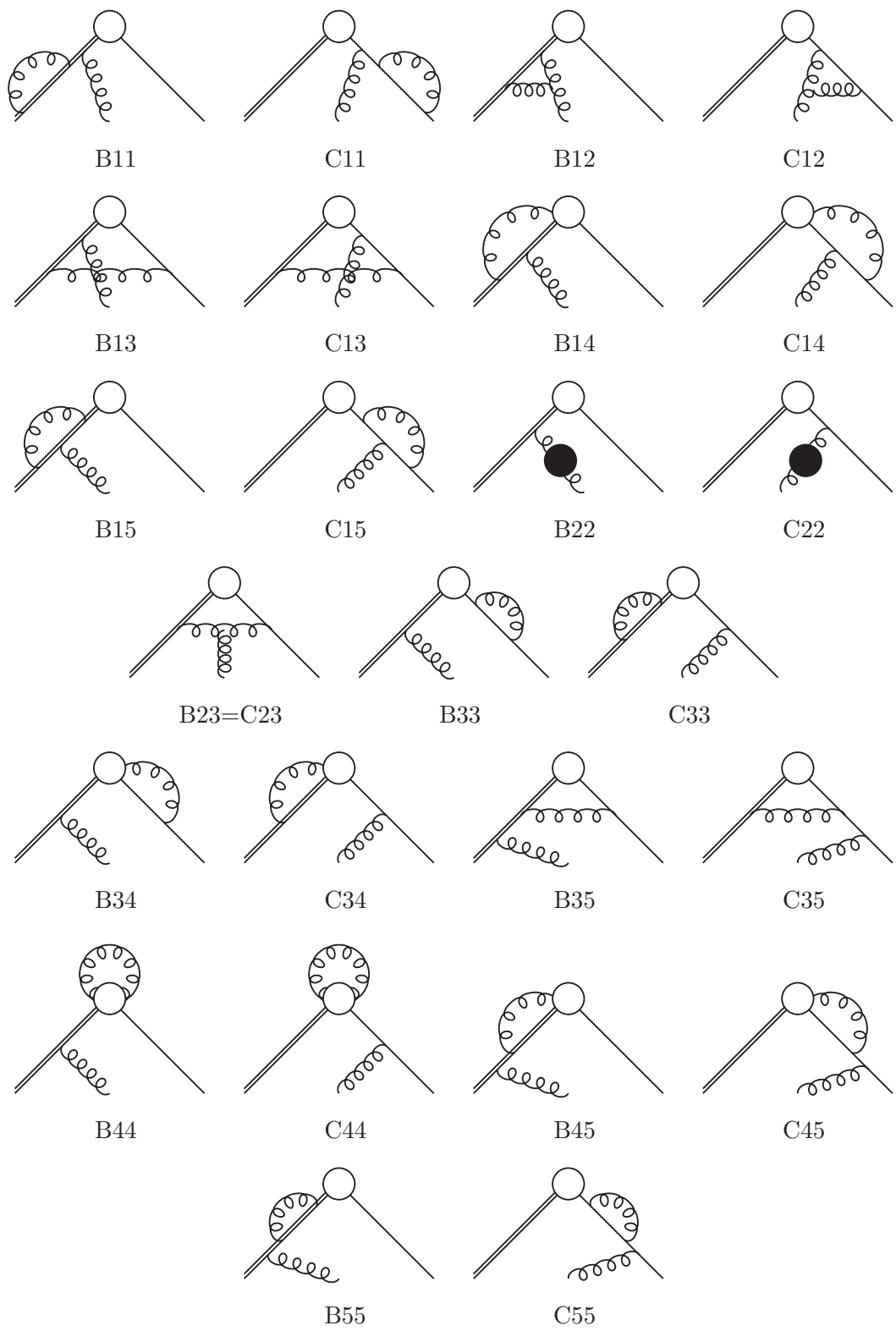
The remaining diagrams yield different contributions for  $\phi_+$  and  $\phi_-$ . For  $\phi_+$  we have the following contributions:

$$\begin{aligned}
 (A13_+) &= 0 \\
 (A23_+) &= \frac{\alpha_s}{4\pi} C_A \frac{(g_s \epsilon_+)}{2} \bar{v} \not{k}_+ \Gamma T^a u \frac{1}{\varepsilon} \left[ \int_{-q_+}^0 \frac{dl_+}{q_+} (k_+ - q_+ - 2l_+) - 2 \int_0^{k_+} \frac{dl_+}{k_+} (k_+ - l_+) \right] \\
 &\quad \times \frac{1}{(k_+ + q_+)(l_+ + q_+)} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+ + l_+) \} \\
 (B23_+) &= 0 \\
 (C13_+) &= 0 \\
 (C14_+) &= -\frac{\alpha_s}{4\pi} (C_A - 2C_F) g_s \frac{1}{\varepsilon} \bar{v} \not{k}_+ \Gamma T^a u \epsilon_+ \left[ \int_0^{k_+} \frac{dl_+}{k_+} + \int_{k_+}^{k_++q_+} \frac{dl_+}{q_+} \frac{k_+ + q_+ - l_+}{l_+} \right] \\
 &\quad \times \frac{1}{k_+ + q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+ - q_+ + l_+) \} \\
 (C45_+) &= C \otimes Z_+|_{\text{LO}}
 \end{aligned}$$

### 3.3 $\phi_-$

For  $\phi_-$ , the remaining diagrams yield the following contributions:

$$\begin{aligned}
 (A13_-) &= \frac{\alpha_s}{4\pi} (C_A - 2C_F) g_s \epsilon_+ \bar{v} \not{k}_- \Gamma T^a u \frac{1}{\varepsilon} \\
 &\quad \times \int_0^{k_+} dl_+ \frac{1}{q_+ k_+} [ \delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ + l_+) ]
 \end{aligned}$$



**Figure 5.** One-loop diagrams obtained from diagrams B and C.

$$\begin{aligned}
 (A23_-) &= \frac{\alpha_s}{4\pi} C_A \frac{1}{2} g_s \frac{1}{\varepsilon} \\
 &\times \left\{ \epsilon_+ \bar{v} \not{\epsilon}_\perp \not{\eta}_- \Gamma T^a u \left[ \int_0^{k_+} \frac{dl_+}{k_+} \left( l_+ - q_+ - 2k_+ \left( 1 + \frac{k_+ - l_+}{k_+ + q_+} \right) \right) \right. \right. \\
 &\quad \left. \left. + \int_{-q_+}^0 \frac{dl_+}{q_+} \left( l_+ + k_+ - 2k_+ \frac{l_+ + q_+}{k_+ + q_+} \right) \right] \right. \\
 &\quad + \epsilon_+ \bar{v} \not{\not{q}}_\perp \not{\eta}_+ \not{\eta}_- \Gamma T^a u \left[ - \int_0^{k_+} \frac{dl_+}{k_+} \frac{l_+ + q_+}{k_+ + q_+} + \int_{-q_+}^0 \frac{dl_+}{q_+} \frac{(l_+ + q_+)(k_+ + 2q_+)}{q_+(k_+ + q_+)} \right] \\
 &\quad \left. + \frac{1}{2} \bar{v} \not{\epsilon}_\perp \not{\eta}_+ \not{\eta}_- \Gamma T^a u \left[ \int_0^{k_+} \frac{dl_+}{k_+} (q_+ + l_+ - 2k_+) - \int_{-q_+}^0 \frac{dl_+}{q_+} (3q_+ + l_+) \right] \right\} \\
 &\quad \times \frac{1}{(k_+ + q_+)(q_+ + l_+)} [\delta(\omega - k_+ - q_+) - \delta(\omega - k_+ + l_+)] \\
 (B23_-) &= -\frac{\alpha_s}{4\pi} \frac{C_A}{2} g_s \bar{v} [\epsilon_+ \not{\eta}_- - \frac{1}{2} \not{\epsilon}_\perp \not{\eta}_+ \not{\eta}_-] \Gamma T^a u \frac{1}{\varepsilon} \\
 &\quad \times \left[ \int_{-q_+}^0 dl_+ \frac{1}{q_+(q_+ + k_+)} - \int_0^{k_+} dl_+ \frac{1}{k_+(q_+ + k_+)} \right] \delta(\omega - k_+ + l_+) \\
 (C13_-) &= \frac{\alpha_s}{4\pi} (C_A - 2C_F) g_s \epsilon_+ \bar{v} \not{\eta}_- \Gamma T^a u \frac{1}{\varepsilon} \\
 &\quad \times \left[ \int_{-q_+}^0 dl_+ \frac{1}{q_+(q_+ + k_+)} - \int_0^{k_+} dl_+ \frac{1}{k_+(q_+ + k_+)} \right] \delta(\omega - q_+ - l_+) \\
 (C14_-) &= \frac{\alpha_s}{4\pi} (C_A - 2C_F) g_s \frac{1}{\varepsilon} \\
 &\quad \times \left\{ \epsilon_+ \bar{v} \not{\eta}_- \Gamma T^a u \left[ \int_0^{k_+} \frac{dl_+}{k_+} \left( l_+ - k_+ \frac{k_+ - l_+}{k_+ + q_+} \right) - \int_{-q_+}^0 \frac{dl_+}{q_+} k_+ \frac{l_+ + q_+}{k_+ + q_+} \right] \right. \\
 &\quad + \frac{1}{2} \epsilon_+ \bar{v} \not{\not{q}}_\perp \not{\eta}_+ \not{\eta}_- \Gamma T^a u \left[ \int_0^{k_+} \frac{dl_+}{k_+} \frac{k_+ - l_+}{k_+ + q_+} + \int_{-q_+}^0 \frac{dl_+}{q_+} \left( \frac{l_+}{q_+} + \frac{l_+ + q_+}{k_+ + q_+} \right) \right] \\
 &\quad \left. + \frac{1}{2} \bar{v} \not{\epsilon}_\perp \not{\eta}_+ \not{\eta}_- \Gamma T^a u \left[ \int_0^{k_+} \frac{dl_+}{k_+} l_+ - \int_{-q_+}^0 \frac{dl_+}{q_+} l_+ \right] \right\} \\
 &\quad \times \frac{1}{(k_+ - l_+)(k_+ + q_+)} [\delta(\omega - k_+ - q_+) - \delta(\omega - q_+ - l_+)] \\
 (C45_-) &= C \otimes Z_-|_{\text{LO}} - 2 \frac{\alpha_s}{4\pi} C_F g_s \bar{v} [\epsilon_+ \not{\eta}_- + \frac{1}{2} \not{\epsilon}_\perp \not{\eta}_+ \not{\eta}_-] \Gamma T^a u \frac{1}{\varepsilon} \\
 &\quad \times \int_0^{k_+ + q_+} \frac{dl_+}{l_+} \frac{k_+ + q_+ - l_+}{(k_+ + q_+)^2} [\delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ - q_+)]
 \end{aligned}$$

(C45<sub>-</sub>) contains an additional contribution compared to the two-parton case, because the determination of  $Z_-|_{\text{LO}}$  relied on the fact that the light-quark coming out of the vertex was on shell, which is not the case for (C45) in the three-parton case.

## 4 One-loop mixing of $\phi_{\pm}$ with 3-parton distribution amplitudes

### 4.1 $\phi_+$

Having calculated the divergent part of all possible diagrams the renormalisation matrix can be determined in a similar manner to ref. [17]. We write:

$$Z_{\pm}(\omega, \omega'; \mu) = \delta(\omega - \omega') + \frac{\alpha_s(\mu)}{4\pi} z_{\pm}^{(1)}(\omega, \omega'; \mu) \quad Z_{\pm,3}(\omega, \omega', \xi'; \mu) = \frac{\alpha_s(\mu)}{4\pi} z_{3\pm}^{(1)}(\omega, \omega', \xi'; \mu), \quad (4.1)$$

with  $z_{\pm}^{(1)}$  being proportional to  $C_F$ . One can schematically write for the matrix element of the bare operator up to one loop:

$$\langle 0 | O_{\pm}(\omega) | H \rangle^{\text{bare}} = Z_h^{1/2} Z_q^{1/2} Z_3^{1/2} Z_g [A + B + C]^{\text{bare}} \quad (4.2)$$

$$+ [B34 + B35 + B45 + C34 + C35 + C45] + [B12 + B15 + C12 + C15 + B55 + C55] \\ + [A12 + A13 + A14 + A23 + A24 + A34 + B23 + C13 + C14]$$

$$= [A + B + C]^{\text{ren}}(\mu) \quad (4.3)$$

$$+ \frac{\alpha_s}{4\pi} \int d\omega' z_{\pm}^{(1)}(\omega, \omega'; \mu) [A + B + C](\omega') + \frac{\alpha_s}{4\pi} \int d\omega' d\xi' z_{3\pm}^{(1)\mu}(\omega, \omega', \xi'; \mu) A_{3\mu}(\omega', \xi'),$$

where the renormalisation constants  $Z_h$ ,  $Z_q$ ,  $Z_3$  and  $Z_g$  come from the heavy-quark, light-quark and gluon external legs and the coupling constant respectively in the leading order contribution. Since the matrix element of the renormalized operator  $O_{\pm}(\omega; \mu)$  must stay finite for  $\varepsilon \rightarrow 0$  and since we know  $z_{\pm}^{(1)}(\omega, \omega'; \mu)$ , we can determine  $z_{3\pm}^{(1)}(\omega, \omega', \xi'; \mu)$  from the poles of the diagrams listed in (4.2).

In the case of  $B$  and  $C$ , the diagrams  $(B34)$ ,  $(B35)$ ,  $(B45)$  and  $(C34)$ ,  $(C35)$ ,  $(C45)$  together with the fermion wave function renormalisation  $Z_h^{1/2}$  and  $Z_q^{1/2}$ , given in eqs. (2.16)–(2.17), add up as indicated above to  $B \otimes z_{\pm}^{(1)}$  and  $C \otimes z_{\pm}^{(1)}$  respectively. The combination of the renormalisation constant for the coupling constant and the gluon field tensor is:

$$Z_3^{1/2} Z_g = 1 - \frac{\alpha_s C_A}{4\pi\varepsilon} \quad (4.4)$$

so that its contribution multiplied by  $B$  and  $C$  cancels the  $C_A$ -part of  $(B12) + (B15)$  and  $(C12) + (C15)$ , whereas the  $C_F$ -part of the same diagrams is cancelled by  $(B55)$  and  $(C55)$ , as expected from general arguments on the renormalisation of the quark-gluon vertex.

The remaining diagrams in eq. (4.2) must be added, and one has to subtract  $A \otimes z_{\pm}^{(1)}$  to extract the three-parton contribution, which amounts to subtracting:

$$A \otimes Z_{\pm}|_{\text{HO}} = 2 \frac{\alpha_s C_F}{4\pi} \Gamma\left(\frac{1}{\varepsilon}\right) \times (-g_s) \frac{\epsilon_+}{q_+} \bar{v} \not{l}_{\pm} \Gamma T^a u \int_0^{\infty} \frac{dl_+}{l_+} \left(\frac{l_+^2}{\mu^2}\right)^{-\varepsilon} \\ \times [\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+ - q_+) - \delta(\omega - k_+ - l_+) + \delta(\omega - k_+)] \quad (4.5)$$

$$A \otimes Z_{\pm}|_{\text{LO}} = 2 \frac{\alpha_s C_F}{4\pi} \Gamma\left(\frac{1}{\varepsilon}\right) \times g_s \frac{\epsilon_+}{q_+} \bar{v} \not{l}_{\pm} \Gamma T^a u \quad (4.6) \\ \times \left[ \int_0^{q_+} \frac{dl_+}{l_+ - q_+} \frac{l_+ + k_+}{q_+ + k_+} [\delta(\omega - k_+ - l_+) - \delta(\omega - k_+ - q_+)] \right]$$

$$\begin{aligned}
 & + \int_{-k_+}^0 \frac{dl_+}{l_+ - q_+} \frac{l_+ + k_+}{q_+ + k_+} [\delta(\omega - k_+ - l_+) - \delta(\omega - k_+ - q_+)] \\
 & - \int_{-k_+}^0 \frac{dl_+}{l_+} \frac{l_+ + k_+}{k_+} [\delta(\omega - k_+ - l_+) - \delta(\omega - k_+)] \\
 A \otimes Z_-|_{\text{HL}} = & 2 \frac{\alpha_s C_F}{4\pi} \frac{1}{\epsilon} \times (-g_s) \frac{\epsilon_+}{q_+} \bar{v} \gamma \not{k}_- \Gamma T^a u \left[ \int_0^{q_+} dl_+ \frac{1}{q_+ + k_+} \delta(\omega - k_+ - l_+) \right. \\
 & \left. + \int_{-k_+}^0 dl_+ \frac{-q_+}{k_+(k_+ + q_+)} \delta(\omega - k_+ - l_+) \right] \quad (4.7)
 \end{aligned}$$

$$A \otimes Z_+|_{\text{HL}} = 0 \quad (4.8)$$

Let us focus on  $\phi_+$  in the remaining part of this section. For the part proportional to  $C_F$ , the Sudakov-like contribution of (A14) matches that of  $Z_{\text{HO}}$  and more generally, an explicit computation shows that the diagrams in the fourth bracket of eq. (4.2) add up exactly to the contribution from  $z_+^{(1)}$  (which is proportional to  $C_F$ ).

For the  $C_A$  part, the contribution from (A14) may seem surprising at first glance, since it seems to involve another Sudakov-like integral, with a double pole in  $1/\epsilon$  generated by the integration of  $l_+$  up to infinity. But let us split the first integral in the  $C_A$  term of (A14) in two intervals, from 0 to  $q_+$  and from  $q_+$  to  $\infty$ , perform a change of variable  $l_+ \rightarrow l_+ + q_+$  and add the second integral, we obtain:

$$\begin{aligned}
 & \frac{1}{\epsilon} \int_0^\infty dl_+ \left[ \left( \frac{l_+}{\mu} \right)^{2\epsilon} - \left( \frac{l_+ + q_+}{\mu} \right)^{2\epsilon} \right] \frac{1}{l_+ + q_+} [\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+)] \\
 & - \frac{1}{\epsilon} \int_0^{q_+} dl_+ \left( \frac{l_+}{\mu} \right)^{2\epsilon} \frac{1}{l_+} [\delta(\omega - k_+ - l_+) - \delta(\omega - k_+)] \quad (4.9)
 \end{aligned}$$

One can see that the first integral yields no pole in  $1/\epsilon$ , and the second integral provides a simple pole. Summing up all the contributions proportional to  $C_A$  (including that of the renormalisation constants from the gluon field tensor and the strong coupling constant), one observes that they cancel exactly.

In summary, the determination of the renormalisation properties of  $\phi_+$  at one loop with a three-parton external state yields a  $C_F$  term equal to the self-mixing obtained from the consideration of two-parton external state, and no  $C_A$  term. This shows that  $\phi_+$  mixes only with itself, and not with three-parton distribution amplitudes, up to one loop.

The fact that  $\phi_+$  occurs in most of the factorisation analyses for B-meson decays suggests that it holds a special status with respect to other  $B$ -meson distribution amplitudes, which is somehow confirmed by our finding of an absence of mixing. For light mesons, conformal symmetry would naturally explain the absence of mixing between distribution with different parton numbers and thus different twists [9]. In the heavy-quark case, conformal symmetry cannot be invoked anymore [33], but our result may be the hint of another symmetry singling out  $\phi_+$  with respect to other distribution amplitudes and explaining that  $\gamma_{+,3} = 0$ .

## 4.2 $\phi_-$

We consider now  $\phi_-$ . One can follow the same argument as before, with a similar pattern of cancellation for the diagrams yielding the same results for  $\phi_+$  and  $\phi_-$ . In particular, one recovers easily the contribution proportional to  $Z_-$ , i.e. the contribution from self-mixing derived from the two-particle case.

But the diagrams do not cancel completely, and there remains a genuine three-particle contribution:

$$\begin{aligned}
 \langle 0|O_-(\omega)|H\rangle^{\text{bare}} &= [A + B + C]^{\text{ren}}(\mu) + \frac{\alpha_s}{4\pi} \int d\omega' z_{\pm}^{(1)}(\omega, \omega'; \mu) [A + B + C](\omega') \\
 &+ \frac{1\alpha_s}{24\pi} g_s \frac{1}{\varepsilon} [q_+ \bar{v}(k) [\not{\epsilon}_\perp(q) \not{\eta}_+ \not{\eta}_- \Gamma T^a] u(p) - \bar{v}(k) [\not{\not{q}}_\perp \not{\eta}_+ \not{\eta}_- \Gamma T^a] u(p) \epsilon_+(q)] \\
 &\times \left\{ (C_A - 2C_F) \left[ \frac{1}{q_+^2} \int_{k_+}^{k_++q_+} dl_+ \left( \frac{1}{l_+} - \frac{1}{k_+} \right) + \frac{1}{(k_++q_+)^2} \int_0^{k_++q_+} \frac{dl_+}{k_+} \right] \right. \\
 &\quad \left. - C_A \frac{1}{k_+} \left[ \frac{1}{(k_++q_+)^2} \int_0^{k_++q_+} dl_+ - \frac{1}{q_+^2} \int_0^{q_+} dl_+ \right] \right\} \\
 &\quad \times \{ \delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ - q_+) \}, \quad (4.10)
 \end{aligned}$$

from which  $z_{-,3}^{(1)\mu}$  can be extracted. If we separate the Dirac structure

$$z_{-,3}^{(1)\mu}(\omega, \omega', \xi'; \mu) = z_{-,3}^{(1)}(\omega, \omega', \xi'; \mu) \gamma_\perp^\mu \not{\eta}_+ \not{\eta}_- \quad (4.11)$$

we obtain

$$\begin{aligned}
 z_{-,3}^{(1)}(\omega, \omega', \xi'; \mu) &= z_{-,3}^{(1)}(\omega, \omega', \xi') \\
 &= -\frac{i}{2\varepsilon} \left[ \frac{\Theta(\omega)}{\omega'} \left\{ (C_A - 2C_F) \left[ \frac{1}{\xi'^2} \frac{\omega - \xi'}{\omega' + \xi' - \omega} \Theta(\xi' - \omega) + \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} \right] \right. \right. \\
 &\quad \left. \left. - C_A \left[ \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} - \frac{1}{\xi'^2} (\Theta(\omega - \omega') - \Theta(\omega - \omega' - \xi')) \right] \right\} \right]_+. \quad (4.12)
 \end{aligned}$$

We defined the  $+$ -distribution as:

$$\left[ f(\omega, \omega', \xi') \right]_+ = f(\omega, \omega', \xi') - \delta(\omega - \omega' - \xi') \int d\omega f(\omega, \omega', \xi') \quad (4.13)$$

Inserting  $\gamma_\perp^\mu \not{\eta}_+ \not{\eta}_-$  into the definition of the three-particle distribution amplitudes one obtains the following expression

$$O_3(\omega', \xi') = 2(2 - D) (\Psi_A(\omega', \xi') - \Psi_V(\omega', \xi')), \quad (4.14)$$

which is exactly the combination arising in the constraint derived from the equation of motion of the light quark in ref. [38]. At order  $\alpha_s$ , the other three-particle distribution amplitudes do not mix with  $\phi_-$ .

One may use that to order  $\alpha_s$  the following relations hold [17]:

$$\frac{\partial O_-(\omega; \mu)}{\partial \log \mu} = -\int d\omega' \frac{\partial Z_-(\omega, \omega'; \mu)}{\partial \log \mu} O_-(\omega'; \mu) - \int d\omega' d\xi' \frac{\partial Z_{-,3}(\omega, \omega', \xi'; \mu)}{\partial \log \mu} O_3(\omega', \xi'; \mu), \quad (4.15)$$



$$\frac{\partial \phi_-(\omega; \mu)}{\partial \log \mu} = -\frac{\alpha_s(\mu)}{4\pi} \left( \int d\omega \gamma_-^{(1)}(\omega, \omega'; \mu) \phi_-(\omega'; \mu) + \int d\omega' d\xi' \gamma_{-,3}^{(1)}(\omega, \omega', \xi'; \mu) [\Psi_A - \Psi_V](\omega', \xi'; \mu) \right)$$

and

$$\frac{\partial Z_{-,3}(\omega, \omega', \xi'; \mu)}{\partial \log \mu} = -2\varepsilon \frac{\alpha_s(\mu)}{4\pi} z_{-,3}^{(1)}(\omega, \omega', \xi'). \quad (4.16)$$

taking into account that  $Z_{-,3}$  starts only at  $O(\alpha_s)$ . This leads to the anomalous dimension  $\gamma_{-,3}^{(1)}$ :

$$\begin{aligned} \gamma_{-,3}^{(1)}(\omega, \omega', \xi'; \mu) &= \gamma_{-,3}^{(1)}(\omega, \omega', \xi') \\ &= 4 \left[ \frac{\Theta(\omega)}{\omega'} \left\{ (C_A - 2C_F) \left[ \frac{1}{\xi'^2} \frac{\omega - \xi'}{\omega' + \xi' - \omega} \Theta(\xi' - \omega) + \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} \right] \right. \right. \\ &\quad \left. \left. - C_A \left[ \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} - \frac{1}{\xi'^2} (\Theta(\omega - \omega') - \Theta(\omega - \omega' - \xi')) \right] \right\} \right]_+ \end{aligned} \quad (4.17)$$

corresponding to the one-loop mixing between  $\phi_-$  and  $\Psi_A - \Psi_V$ .

## 5 Calculation in a general covariant gauge

We have computed the mixing between gauge-invariant operators, and we could in principle have chosen any gauge to perform our determination of the renormalisation properties of the latter. We can check the validity of our previous computations by computing the nontrivial diagrams considered previously in a general covariant gauge, where we replace the Feynman-gauge gluon-propagator by

$$d_{\mu\nu}^{ab}(k) = -i\delta^{ab} \left[ g_{\mu\nu} - (1 - \alpha) \frac{k_\mu k_\nu}{k^2} \right] \quad (5.1)$$

Our result should be gauge invariant, so that the parts proportional to  $(1 - \alpha)$  should cancel.

### 5.1 Two-parton external state and $Z_\pm$

First, we can repeat the computation of refs. [34, 35], recalled in section 2, for the case of a two-parton external state. The gauge dependent part is:

$$M_{HO\pm} \rightarrow -ig_s^2 C_F (1 - \alpha) \int \frac{d^4 l}{(2\pi)^4} \bar{v} \not{l}_\pm \Gamma T^a u \frac{1}{l^4} \{ \delta(\omega - k_+ - l_+) - \delta(\omega - k_+) \}, \quad (5.2)$$

$$M_{LO\pm} \rightarrow -ig_s^2 C_F (1 - \alpha) \int \frac{d^4 l}{(2\pi)^4} \bar{v} \not{l}_\pm \Gamma T^a u \frac{1}{l^4} \{ \delta(\omega - k_+ + l_+) - \delta(\omega - k_+) \}, \quad (5.3)$$

$$M_{HL\pm} \rightarrow ig_s^2 C_F (1 - \alpha) \int \frac{d^4 l}{(2\pi)^4} \bar{v} \not{l}_\pm \Gamma T^a u \frac{1}{l^4} \delta(\omega - k_+ + l_+). \quad (5.4)$$

These integrals should be equipped with an infrared regulator (for instance, a gluon mass  $m_g$ ) in order to ensure that we keep only the ultraviolet divergences of interest here when

we pick up the poles in  $\varepsilon$  (otherwise, dimensional regularisation would treat both ultraviolet and infrared divergences of the integral as poles in  $\varepsilon$ ).

In ref. [39], such integrals were considered with a particular focus on the integration over the different light-cone components  $d^4l \rightarrow \frac{1}{2}dl_+dl_-d^2l_\perp$ . Let us suppose that we want to integrate over  $l_-$ . There is a single pole, at  $l_- = (m_g^2 - i\epsilon + \vec{l}_\perp^2)/l_+$ , that we can always avoid by choosing the contour from above for  $l_+ > 0$  and from below for  $l_+ < 0$ . It seems to indicate that such integral should be 0, which is incorrect. As proposed in ref. [39], a proper regularisation leads to the conclusion that an integration over the minus (plus) component results in a delta-distribution  $\delta(l_+)$  ( $\delta(l_-)$ ), and one gets the equality:

$$\int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \delta(\omega - k_+ \pm l_+) = \delta(\omega - k_+) \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \quad (5.5)$$

We see that  $M_{HO}$  and  $M_{LO}$  vanish, whereas  $M_{HL}$  cancels the gauge-dependent part of the wave-function renormalisation for the heavy and the light quarks:

$$Z_q = 1 + C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} [-1 + (1 - \alpha)] \quad Z_h = 1 + C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon} [2 + (1 - \alpha)] \quad (5.6)$$

Therefore, the gauge-dependent parts of the different contributions cancel and we have checked that the expression of  $Z_\pm$  is indeed gauge independent.

## 5.2 Three-parton external state and $Z_{-3}$

The issue becomes a little more involved if a three-particle state is considered. The complete formulae can be found in appendix B, but we can outline the pattern of cancellation for the gauge-dependent part among the various diagrams.

We can identify the different gauge-dependent contributions in eq. (4.10). The diagrams (A44), (B44) and (C44), which vanish trivially in the Feynman gauge, have to be taken into account, but their contributions can be shown to vanish through eq. (5.5). This is also the case for the diagrams (A14), (A24) and (A34). (B13) and (B14) remain finite as in the Feynman gauge.

In analogy with the two-particle case, the diagrams (B34), (B35), (B45) and (C34), (C35), (C45) cancel with the gauge-dependent part of  $Z_q^{1/2} Z_h^{1/2}$  multiplied by  $B$  and  $C$  respectively. One has to pay attention to (C35) and (C45) that give additional contributions canceling each other. (B12), (B15), (B55) and (C12), (C15), (C55) cancel against the gauge-dependent part of  $Z_3^{1/2} Z_g$  multiplied by  $B$  and  $C$  respectively.

Finally, one is left with (A12), (A13), (A23), (B23), (C13), (C14) and  $Z_h^{1/2} Z_2^{1/2} Z_g Z_3^{1/2} \times A$ . The sum of (C13) and (C14) is finite, and once eq. (5.5) is applied, only the following expressions remain:

$$(A12) = -ig_s^3 \frac{C_A}{4} (1 - \alpha) \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \frac{1}{q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+) \} \bar{v} \not{\eta}_\pm \Gamma T^a u \epsilon_+, \quad (5.7)$$

$$(A13) = ig_s^3 \left( \frac{C_A}{2} - C_F \right) (1 - \alpha) \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \frac{1}{q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+) \} \bar{v} \not{\eta}_\pm \Gamma T^a u \epsilon_+, \quad (5.8)$$

$$(A23) = -ig_s^3 \frac{C_A}{4} (1 - \alpha) \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \frac{1}{q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+) \} \bar{v} \not{\eta}_\pm \Gamma T^a u \epsilon_+, \quad (5.9)$$

$$(B23) = -ig_s^3 \frac{C_A}{4} (1 - \alpha) \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \frac{1}{q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+) \} \bar{v} \not{l}_\pm \Gamma T^a u \epsilon_+. \quad (5.10)$$

Picking up the ultraviolet divergences from the integrals, we obtain finally for the sum:

$$\frac{\alpha_s}{4\pi} \left( \frac{C_A}{4} + C_F \right) (1 - \alpha) \frac{1}{\epsilon} \bar{v} \not{l}_\pm \Gamma T^a u \epsilon_+ \frac{1}{q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+) \} \quad (5.11)$$

which cancels exactly the  $(1 - \alpha)$ -dependent part of the combination

$$Z_h^{1/2} Z_2^{1/2} Z_g Z_3^{1/2} \times A = \left( 1 + \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} \left( \frac{C_F}{2} [1 + 2(1 - \alpha)] - \frac{C_A}{4} [4 - (1 - \alpha)] \right) \right) \times A, \quad (5.12)$$

The calculation for  $\phi_+(\omega)$  is simpler, since the replacement of  $\not{l}_-$  by  $\not{l}_+$  implies the absence of contributions proportional to  $\bar{v} \not{l}_+ \not{l}_- \Gamma T^a u$ . Therefore the additional terms of (C35) and (C45) vanish as well as the gauge-dependent contributions of the diagrams (C13) and (C14). Following the same lines in both cases, one can conclude that there is no gauge-dependence for the renormalisation of  $\phi_+$  and  $\phi_-$ .

## 6 Conclusion

In this paper, we have studied the mixing of both two-particle distribution amplitudes  $\phi_+$  and  $\phi_-$  with three-particle ones up to one-loop. Using the fact that RGE is a short-distance property of the operator, we used matrix elements of the operators with a quark-antiquark-gluon external state. Determining the ultraviolet divergences of the corresponding diagrams allowed us to recover the known one-loop self-mixing of  $\phi_+$  and  $\phi_-$ , but also to determine the role of three-parton distribution amplitudes. We have established that  $\phi_+$  mixes only with itself, whereas  $\phi_-$  does mix with  $[\Psi_A - \Psi_V]$ , and we have provided the corresponding anomalous dimension. Through the use of a general covariant gauge, we have checked that our results were indeed gauge invariant, providing further support to our expressions.

We can relate our results to other comments on the  $B$ -meson distribution amplitudes in the literature. For instance, the fact that  $\phi_+$  does not mix with three-parton distribution amplitudes was already presented in ref. [40]. In this article, a computation similar to ours was sketched in the case of  $\phi_+$ , with the conclusion (presented in eq. (2) of this reference) that the only ultraviolet one-loop divergence for  $O_+$  is proportional to itself, whereas contributions proportional to higher-dimension operators have only infrared divergences.

As mentioned in the introduction, the presence of  $\delta(\omega - \omega') \log(\mu/\omega)$  in the renormalization-matrices provides a radiative tail to  $\phi_\pm$  falling off like  $(\log \omega)/\omega$  for large  $\omega$ . It requires one to consider either negative moments of the distribution amplitudes  $\phi_-$ ,  $\phi_+$ , or positive moments with an ultraviolet cut-off [17, 34, 35, 40, 41]:

$$\langle \omega^N \rangle_\pm(\mu) = \int_0^{\Lambda_{UV}} d\omega \omega^N \phi_\pm(\omega; \mu) \quad (6.1)$$

On the contrary, it is interesting to notice the limit

$$\lim_{\Lambda_{UV} \rightarrow \infty} \int_0^{\Lambda_{UV}} d\omega \omega^N z_{-,3}^{(1)}(\omega, \omega', \xi') = 0 \quad N = 0, 1 \quad (6.2)$$

This is relevant for the calculation of the three-particle contributions to the moments:

$$\int_0^{\Lambda_{UV}} d\omega \omega^N \phi_-(\omega; \mu) = 1 + \frac{\alpha_s}{4\pi} \left( \int d\omega' \phi_-(\omega') \int_0^{\Lambda_{UV}} d\omega \omega^N z_-^{(1)}(\omega, \omega'; \mu) \right. \\ \left. - \int d\omega' d\xi' (2-D) [\Psi_A - \Psi_V](\omega', \xi') \int_0^{\Lambda_{UV}} d\omega \omega^N z_{-,3}^{(1)}(\omega, \omega', \xi') \right) \quad (6.3)$$

Therefore, there is no contribution to the two lowest moments of  $\phi_-$  from three-particle distribution amplitudes, which confirms the statement made after eq. (62) in ref. [35]. We have explicitly checked that this property of  $z_{-,3}^{(1)}$  does not hold for higher positive moments ( $N \geq 2$ ).

In ref. [38] (see also refs. [42–44]) were derived two different relations between  $\phi_+$  and  $\phi_-$  on one hand and the four three parton-distribution amplitudes  $\Psi_V, \Psi_A, X_A$  and  $Y_A$  on the other hand:

$$\omega \phi'_-(\omega) + \phi_+(\omega) = I(\omega) \quad (6.4)$$

$$(\omega - 2\bar{\Lambda})\phi_+(\omega) + \omega\phi_-(\omega) = J(\omega) \quad (6.5)$$

$I$  ( $J$ ) is an integro-differential expression involving  $\Psi_A - \Psi_V$  ( $\Psi_A + X_A$  and  $\Psi_V$ ). The first relation comes from the equation of motion for the light quark and the latter one from the heavy quark (as suggested by the presence of the HQET parameter  $\bar{\Lambda} = M_B - m_b$ ). The use of the equation of motion of the heavy quark was criticised in ref. [33, 35], because this is linked to the heavy-quark limit which does not commute with the light-cone limit. Moreover, this equation can be derived only if one leaves the light-cone limit, which is not needed for the first relation. One can notice that the shapes of the distribution amplitudes have been derived in the Wandura-Wilczek approximation, where three-parton distribution amplitudes are neglected ( $I = J = 0$ ), leading to rather unphysical shapes for the distribution amplitudes. If we assume that the three-particle distribution amplitudes mix separately into themselves [45, 46], our work shows that the renormalisation-scale dependence of eq. (6.4), derived from the light-quark equation of motion, is satisfying: by applying  $d/d \log \mu$  to the equation of motion, both  $\phi_-$  and  $\Psi_A - \Psi_V$  yield a term proportional to  $\Psi_A - \Psi_V$ . The contributions proportional to the two-particle distribution amplitudes were shown to cancel in the Wandura-Wilczek approximation in appendix D of ref. [35] (eq. (6.4) was also shown to hold in a specific non-relativistic model for  $\phi_+, \phi_-, \Psi_A, \Psi_V$  in appendix C of the same reference). Eq. (6.5) does not seem to have such a satisfactory renormalisation-scale dependence, which would add to the various criticisms raised against this equation (see ref. [45] for further discussion of this issue).

More generally, the influence of three-particle distribution-amplitudes on  $\phi_-(\omega; \mu)$  requires one to model them. However, the only available models [31] assume  $\Psi_A(\omega, \xi) = \Psi_V(\omega, \xi)$  and they yield no contribution to the evolution of  $\phi_-(\omega; \mu)$ . For practical calculations as well as for further model-building of distribution amplitudes beyond  $\phi_+$ , one needs the evolution kernel of the three-particle distribution amplitudes, which will be the subject of a future work [46].

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## A Extraction of poles related to UV divergences

In this paper, we compute various integrals in dimensional regularisation to extract the  $\varepsilon$ -poles related to UV divergences. The textbook procedure consists in a covariant analysis, where all space directions are treated on the same footing. Since we use light-cone coordinates with a privileged direction for the definition and discussion of the distribution amplitudes, we use a slightly less usual method which we apply on the illustrative integral:

$$I = \int \frac{d^4 l}{(2\pi)^4} f(l_+) \frac{1}{l^2} \frac{1}{(l-k)^2} \quad (\text{A.1})$$

where  $f$  is an arbitrary function of  $l_+$  alone, corresponding to a gluon line (of momentum  $l$ ) attached to a light-quark line (of incoming momentum  $k$ ). Such an integral is needed already to compute the mixing of  $\phi_{\pm}$  into themselves ( $Z_{\text{LO}}$ ). We want to perform the integrals over  $l_-$  and  $\vec{l}_{\perp}$  in  $4 - 2\varepsilon$  dimensions and isolate the poles in  $\varepsilon$  related to UV divergences. We therefore introduce a small mass  $m$  for the light quark to regularise (soft) IR divergences that are of not interest for the determination of the RG properties of the distribution amplitudes.

We perform first the integral over  $l_-$  by identifying the poles in the complex  $l_-$  plane:

$$I = \int \frac{dl_+ dl_- d^2 \vec{l}_{\perp}}{2(2\pi)^4} f(l_+) \frac{1}{l^2 + i0^+} \frac{1}{l^2 - 2k \cdot l + i0^+} \quad (\text{A.2})$$

which are  $l_- = (\vec{l}_{\perp}^2 - i0^+)/l_+$  and  $l_- = k_- + ((\vec{l}_{\perp} - \vec{k}_{\perp})^2 - i0^+)/l_+$ , whose positions with respect to the real axis depend on the value of  $l_+$ . If  $l_+$  is negative or larger than  $k_+$ , the two poles sit on the same side, and thus the contour integral yields 0. If  $0 < l_+ < k_+$ , the two poles are on different sides and one gets a non-vanishing contribution, for instance by closing the contour in the lower half-plane and thus picking up the first pole (associated with the  $l^2$ -denominator):

$$I = \frac{i}{4\pi} \int_0^{k_+} dl_+ \int \frac{d^2 \vec{l}_{\perp}}{(2\pi)^2} \frac{f(l_+)}{k_- l_+^2 + k_+ \vec{l}_{\perp}^2 - 2\vec{k}_{\perp} \cdot \vec{l}_{\perp} l_+} \quad (\text{A.3})$$

Then one can perform the integral over the  $2 - 2\varepsilon$  transverse dimensions, which yields the result:

$$I = \Gamma(\varepsilon) \frac{i}{(4\pi)^{2-\varepsilon}} \int_0^{k_+} \frac{dl_+}{k_+} f(l_+) \left[ \frac{m^2 l_+^2}{\mu^2 k_+^2} \right]^{-\varepsilon} \quad (\text{A.4})$$

This expression yields a single pole in  $\varepsilon$  corresponding to the UV-divergent part of the integral, which enters  $Z_{\text{LO}}$ . The same procedure is applied to all the diagrams, with sometimes more involved integrals (up to four propagators), leading to the results quoted in the present article.

## B Gauge dependence of the diagrams in a general covariant gauge

In this appendix, we collect the integrals from the different diagrams that are proportional to the gauge parameter  $(1 - \alpha)$ . For the well known cases (B12), (B15), (B55), (C12), (C15) and (C55), corresponding to the vertex renormalisation, the integrals have already been carried out.

$$\begin{aligned}
 (A12) &= -ig_s^3(1 - \alpha)\frac{C_A}{2}\bar{v}\not{l}_\pm\Gamma T^a u\epsilon_+ \\
 &\quad \times \int \frac{d^4l}{(2\pi)^4} \frac{1}{l_+ + q_+} \{\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+)\} \\
 &\quad \times \left[ \left(1 - \frac{l_+ + q_+}{2q_+}\right) \frac{1}{l^4} + \frac{l_+ + q_+}{2q_+} \frac{1}{(l + q)^4} \right] \\
 (A13) &= \frac{i}{2}g_s^3(1 - \alpha)(C_A - 2C_F)\bar{v}\not{l}_\pm\Gamma T^a u\epsilon_+ \\
 &\quad \times \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \frac{1}{q_+} \{\delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ + l_+)\} \\
 (A14) &= ig_s^3(1 - \alpha)\bar{v}\not{l}_\pm\Gamma T^a u\epsilon_+ \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^4} \\
 &\quad \times \left\{ C_F \frac{1}{q_+} (\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+ - q_+)) - \right. \\
 &\quad \left. - \delta(\omega - k_+ - l_+) + \delta(\omega - k_+) \right) \\
 &\quad \left. - \frac{C_A}{2} \frac{1}{q_+} \left( \frac{l_+}{l_+ + q_+} \{\delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+)\} \right. \right. \\
 &\quad \left. \left. - \delta(\omega - k_+ - l_+) + \delta(\omega - k_+) \right) \right\} \\
 (A23) &= -ig_s^3(1 - \alpha)\frac{C_A}{2} \int \frac{d^4l}{(2\pi)^4} \frac{1}{l_+ + q_+} \{\delta(\omega - k_+ - q_+) - \delta(\omega - k_+ + l_+)\} \\
 &\quad \times \left[ \left( \frac{1}{l^4} \left(1 - \frac{1}{2} \frac{l_+ + q_+}{q_+}\right) + \frac{1}{2} \frac{l_+ + q_+}{q_+} \frac{1}{(l + q)^4} \right) \bar{v}\not{l}_\pm\Gamma T^a u\epsilon_+ \right. \\
 &\quad \left. + \frac{1}{2} \frac{l_+ + q_+}{k_+ + q_+} \left( \frac{1}{(k - l)^2(l + q)^2} - \frac{1}{(l + q)^4} \right) \bar{v}\not{l}_\pm\not{l}_\pm\Gamma T^a u \right] \\
 (A24) &= +ig_s^3(1 - \alpha)\frac{C_A}{2}\bar{v}\not{l}_\pm\Gamma T^a u\epsilon_+ \\
 &\quad \times \int \frac{d^4l}{(2\pi)^4} \left[ \frac{1}{(l + q)^4} \left(1 + \frac{l_+}{2q_+}\right) + \frac{l_+}{l^4} \left( \frac{1}{l_+ + q_+} - \frac{1}{2q_+} \right) \right] \\
 &\quad \times \left[ \frac{1}{l_+} \{\delta(\omega - k_+ + l_+) - \delta(\omega - k_+)\} \right. \\
 &\quad \left. + \frac{1}{q_+} \{\delta(\omega - k_+ - q_+) - \delta(\omega - k_+)\} \right] \\
 (A34) &= -ig_s^3(1 - \alpha)\bar{v}\not{l}_\pm\Gamma T^a u\epsilon_+ \\
 &\quad \times \int \frac{d^4l}{(2\pi)^4} \left[ C_F \frac{1}{q_+} (\delta(\omega - k_+ - q_+) + \delta(\omega - k_+ + l_+)) \right. \\
 &\quad \left. - \delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+) \right]
 \end{aligned}$$

$$-\frac{C_A}{2} \left( \frac{1}{l_+ + q_+} \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+ + l_+) \} \right. \\ \left. - \frac{1}{q_+} \{ \delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ + l_+) \} \right)$$

$$(A44) = ig_s^3(1 - \alpha) \bar{v} \not{k}_\pm \Gamma T^a u \epsilon_+$$

$$\times \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \left[ C_F \frac{1}{q_+} \left\{ \delta(\omega - k_+ + l_+) - \delta(\omega - k_+) \right. \right. \\ \left. \left. + \delta(\omega - k_+ - q_+) - \delta(\omega - k_+ - q_+ + l_+) - \frac{l_+}{q_+} (\delta(\omega - k_+ - q_+) - \delta(\omega - k_+)) \right\} \right. \\ \left. + \frac{C_A}{2} \frac{l_+}{q_+} \left\{ \frac{1}{l_+ + q_+} (\delta(\omega - k_+ - q_+) - \delta(\omega - k_+ + l_+)) \right. \right. \\ \left. \left. + \frac{1}{l_+ - q_+} (\delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+)) \right\} \right]$$

$$(B12) = \frac{3C_A}{4} \frac{\alpha_s}{4\pi} g_s(1 - \alpha) \frac{1}{\epsilon} \delta(\omega - k_+) \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(B13) = 0$$

$$(B14) = 0$$

$$(B15) = -\frac{1}{2} (C_A - 2C_F) \frac{\alpha_s}{4\pi} g_s(1 - \alpha) \frac{1}{\epsilon} \delta(\omega - k_+) \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(B23) = -ig_s^3(1 - \alpha) \frac{C_A}{4} \int \frac{d^4 l}{(2\pi)^4} \delta(\omega - k_+ + l_+)$$

$$\times \left[ \frac{1}{q_+} \left( \frac{1}{(l_+ + q_+)^4} - \frac{1}{l_+^4} \right) \bar{v} \not{k}_\pm \Gamma T^a u \epsilon_+ \right. \\ \left. + \frac{1}{k_+ + q_+} \left( \frac{1}{(k_+ - l_+)^2 (l_+ + q_+)^2} - \frac{1}{(l_+ + q_+)^4} \right) \right] \bar{v} \not{\epsilon} \not{k}_+ \not{k}_\pm \Gamma T^a u$$

$$(B34) = ig_s^3(1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \{ \delta(\omega - k_+ + l_+) - \delta(\omega - k_+) \} \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(B35) = -ig_s^3(1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \delta(\omega - k_+ + l_+) \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(B44) = -ig_s^3(1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \{ \delta(\omega - k_+ + l_+) - \delta(\omega - k_+) \} \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(B45) = ig_s^3(1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \{ \delta(\omega - k_+ - l_+) - \delta(\omega - k_+) \} \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(B55) = -C_F \frac{\alpha_s}{4\pi} g_s(1 - \alpha) \frac{1}{\epsilon} \delta(\omega - k_+) \bar{v} \not{k}_\pm \Gamma T^a u \frac{v \cdot \epsilon}{v \cdot q}$$

$$(C12) = -\frac{3C_A}{4} \frac{\alpha_s}{4\pi} g_s(1 - \alpha) \frac{1}{\epsilon} \delta(\omega - k_+ - q_+) \frac{1}{(k_+ + q_+)^2} \bar{v} \not{\epsilon} (\not{k}_+ + \not{q}) \not{k}_\pm \Gamma T^a u$$

$$(C13) = \frac{i}{2} g_s^3(1 - \alpha) (C_A - 2C_F) \bar{v} \not{\epsilon} \not{k}_+ \not{k}_\pm \Gamma T^a u$$

$$\times \int \frac{d^4 l}{(2\pi)^4} \frac{1}{k_+ + q_+} \delta(\omega - k_+ - q_+ + l_+) \left[ \frac{1}{l^2 (k_+ + q_+ - l_+)^2} - \frac{1}{l^4} \right]$$

$$(C14) = \frac{i}{2} g_s^3 (1 - \alpha) (C_A - 2C_F) \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 (k + q - l)^2} \frac{1}{k_+ + q_+} \\ \times \{ \delta(\omega - k_+ - q_+) - \delta(\omega - k_+ - q_+ + l_+) \} \bar{v} \not{\epsilon} \not{k}_+ \not{k}_\pm \Gamma T^a u$$

$$(C15) = \frac{1}{2} (C_A - 2C_F) \frac{\alpha_s}{4\pi} g_s (1 - \alpha) \frac{1}{\varepsilon} \delta(\omega - k_+ - q_+) \frac{1}{(k_+ + q_+)^2} \bar{v} \not{\epsilon} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u$$

$$(C34) = -i g_s^3 (1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \frac{1}{(k_+ + q_+)^2} \bar{v} \not{\epsilon} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u \\ \times \{ \delta(\omega - k_+ - q_+ - l_+) - \delta(\omega - k_+ - q_+) \}$$

$$(C35) = i g_s^3 (1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \left[ \frac{1}{l^4} \frac{1}{(k_+ + q_+)^2} \bar{v} \not{\epsilon} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u \right. \\ \left. + \frac{1}{2} \left[ \frac{1}{l^2 (k_+ + q_+ - l)^2} - \frac{1}{l^4} \right] \frac{1}{k_+ + q_+} \bar{v} \not{\epsilon} \not{k}_+ \not{k}_\pm \Gamma T^a u \right] \delta(\omega - k_+ - q_+ + l_+)$$

$$(C44) = i g_s^3 (1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \{ \delta(\omega - k_+ + l_+) - \delta(\omega - k_+) \} \bar{v} \not{\epsilon} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u \frac{1}{(k_+ + q_+)^2}$$

$$(C45) = -i g_s^3 (1 - \alpha) C_F \int \frac{d^4 l}{(2\pi)^4} \left[ \frac{1}{l^4} \frac{1}{(k_+ + q_+)^2} \bar{v} \not{\epsilon} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u \right. \\ \left. + \frac{1}{2} \left[ \frac{1}{l^2 (k_+ + q_+ - l)^2} - \frac{1}{l^4} \right] \frac{1}{k_+ + q_+} \bar{v} \not{\epsilon} \not{k}_+ \not{k}_\pm \Gamma T^a u \right] \\ \times \{ \delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ - q_+) \}$$

$$(C55) = C_F \frac{\alpha_s}{4\pi} g_s (1 - \alpha) \frac{1}{\varepsilon} \delta(\omega - k_+ - q_+) \frac{1}{(k_+ + q_+)^2} \bar{v} \not{\epsilon} (\not{k} + \not{q}) \not{k}_\pm \Gamma T^a u$$

As mentioned at the end of section 6, the terms proportional to  $\not{\epsilon} \not{k}_+ \not{k}_\pm$  vanish for  $\phi_+$ . In addition, all the integrals must be understood with an infrared regulator, since we are only interested in their ultraviolet behaviour.

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